

On Spherical Neutrosophic SuperHyperGraphs And Complex Spherical Neutrosophic SuperHyperGraphs

T. Fujita

Independent Researcher, Tokyo, Japan.

E-mail: Takaaki.fujita060@gmail.com

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Abstract

Graph theory offers a fundamental framework for representing and analyzing relationships among entities via vertices and edges. Hypergraphs extend this model by employing hyperedges that can connect multiple vertices simultaneously, enabling the capture of higher-order interactions. Superhypergraphs generalize hypergraphs further by iteratively applying the powerset operation, thus supporting multi-level and self-referential connection structures. In this paper, we introduce spherical-neutrosophic extensions based on spherical neutrosophic graphs by defining *spherical neutrosophic hypergraphs* and *spherical neutrosophic superhypergraphs*, and we investigate their fundamental properties. As an additional result, by incorporating ideas from *complex spherical fuzzy sets*, we define *complex spherical neutrosophic superhypergraphs* and examine their characteristics.

Keywords: *Superhypergraph, Hypergraph, Neutrosophic Graph, Spherical Neutrosophic Graph, Complex Spherical Neutrosophic Graph*

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1 Introduction and Literature Review

1.1 Hypergraphs and Superhypergraphs

A graph is a concept for representing objects and the relationships that connect them [1]. Graph-theoretic ideas have a very broad range of applications and are

widely recognized as an important area of research. However, standard graphs restrict edges to unordered pairs of vertices. Hypergraphs broaden this framework by allowing each hyperedge to connect any nonempty subset of vertices, thereby capturing higher-order relationships [2, 3].

However, to model more deeply hierarchical concepts that cannot be adequately represented by hypergraphs, the notion of a SuperHyperGraph has been introduced in recent years. Superhypergraphs build upon hypergraphs by iteratively applying the powerset operation: at each level, both “supervertices” and “superedges” are drawn from the powerset of the preceding level, resulting in a multi-tiered hierarchy of connections [4, 5]. SuperHyperGraphs have recently attracted considerable attention because they can represent concepts hierarchically more effectively than graphs and hypergraphs [6, 7, 8].

For quick reference, Table 1 summarizes the essential distinctions among graphs, hypergraphs, and n -SuperHyperGraphs.

<i>Aspect</i>	<i>Graph</i>	<i>Hypergraph</i>	<i>n-SuperHyperGraph</i>
Objects	Vertices, edges.	Vertices, hyperedges.	n -supervertices, superhyperedges.
Links connect	Two vertices.	Any nonempty vertex subset.	Nonempty supervertex subsets (with nested supervertices).
Formal model	$G = (V, E), E \subseteq \binom{V}{2}$.	$H = (V, E), E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$.	$\text{SHG}^{(n)} = (V, E), V \subseteq \mathcal{P}^n(V_0), E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$.
Main emphasis	Pairwise relations.	Higher-order relations.	Higher-order <i>and</i> hierarchical (multi-level) relations.

Table 1: Compact comparison of graphs, hypergraphs, and n -SuperHyperGraphs.

1.2 Models for Uncertain Graph-Based Networks

To incorporate uncertainty into graph-based models, several extensions have been proposed. In fuzzy graphs, each vertex and edge is endowed with a membership degree in $[0, 1]$, representing gradual inclusion [9, 10, 11]. Intuitionistic fuzzy graphs further introduce a non-membership degree alongside membership [12], whereas neutrosophic graphs add an indeterminacy component in addition to truth and falsity values [13, 14]. Related notions within neutrosophic graph theory include bipolar neutrosophic graphs [15] and quadripartitioned neutrosophic graphs [16]. Plithogenic graphs extend these ideas by quantifying contradictions among multiple attributes [17]. These uncertainty-aware graph models have also been extended

Table 2: Compact overview of spherical neutrosophic graph / hypergraph / n -superhypergraph models.

<i>Model</i>	<i>Crisp core</i>	<i>Spherical neutrosophic data (unit-ball) + support</i>
SNG	$G^* = (V, E^*)$	Vertex maps $(T_V, I_V, F_V) : V \rightarrow [0, 1]^3$; edge maps $(T_E, I_E, F_E) : V \times V \rightarrow [0, 1]^3$. Supported on $\{x, y\} \in E^*$; dominated by endpoints via min / max.
SNHG	$(V, \mathcal{E}), \mathcal{E} \subseteq \mathcal{P}^*(V)$	Vertex maps (T_V, I_V, F_V) ; incidence maps $(T_{\mathcal{E}}, I_{\mathcal{E}}, F_{\mathcal{E}}) : \mathcal{E} \times V \rightarrow [0, 1]^3$. Supported on $v \in e$; dominated by e 's endpoints via $\min_{u \in e} / \max_{u \in e}$.
SNSHG $_n$	$\text{SHG}^{(n)} = (V, E), V \subseteq \mathcal{P}^n(V_0), E \subseteq \mathcal{P}^*(V)$	Supervertex maps $(T_V, I_V, F_V) : V \rightarrow [0, 1]^3$; superedge incidences $(T_E, I_E, F_E) : E \times V \rightarrow [0, 1]^3$. Supported on $v \in e$; dominated by $\min_{u \in e} / \max_{u \in e}$. ($n = 0 \Rightarrow$ SNHG; $ e = 2 \Rightarrow$ SNG.)

to hypergraphs and SuperHyperGraphs [18, 19], and extensive foundational and applied research has been conducted on such extensions, including applications to decision-support and related domains.

As one such notion, the concept of a spherical neutrosophic graph has been introduced. A spherical neutrosophic graph assigns unit-ball truth, indeterminacy, and falsity degrees to vertices and edges while respecting adjacency constraints [20]. It is also known to generalize the notion of a spherical fuzzy graph [21]. Moreover, related frameworks such as complex fuzzy sets[22], complex neutrosophic sets[23], and complex spherical fuzzy sets[24] have also been studied, and extensive foundational and applied research has been conducted on these notions, including applications to decision-support and other domains.

1.3 Our Contribution

In view of the above, research on SuperHyperGraphs is of considerable importance. In this paper, we introduce spherical-neutrosophic extensions inspired by spherical neutrosophic graphs by defining *spherical neutrosophic hypergraphs* and *spherical neutrosophic superhypergraphs*, and we investigate their fundamental properties. As an additional result, by incorporating ideas from *complex spherical fuzzy sets*, we define *complex spherical neutrosophic superhypergraphs* and examine their characteristics. For reference, a compact overview of spherical neutrosophic graph, hypergraph, and n -superhypergraph models is provided in Table 2.

2 Preliminaries

In this section we fix notation and recall the basic set-theoretic constructions that will be used throughout the paper.

2.1 Hypergraphs and iterated powersets

Classical graph theory generalizes to *hypergraphs* by allowing an edge to join an arbitrary (finite) number of vertices rather than only two. This extension is useful for modeling multi-way relations in various domains; see, e.g., [3, 25].

Definition 2.1 (Powerset and nonempty powerset). For any set S , the *powerset* of S is

$$\mathcal{P}(S) := \{ A \mid A \subseteq S \}.$$

The *nonempty powerset* of S is

$$\mathcal{P}^*(S) := \mathcal{P}(S) \setminus \{\emptyset\}.$$

Definition 2.2 (Hypergraph). A *hypergraph* is a pair $H = (V(H), E(H))$ where

- $V(H)$ is a nonempty finite set (the *vertex set*), and
- $E(H) \subseteq \mathcal{P}^*(V(H))$ is a finite family of nonempty subsets of $V(H)$ (the *hyperedges*).

Thus each hyperedge $e \in E(H)$ is a nonempty set of vertices and may connect two or more vertices simultaneously. See [26, 2] for standard references.

Definition 2.3 (Iterated powersets). Let S be a set and let $n \in \mathbb{N} \cup \{0\}$. Define the iterated powersets $\mathcal{P}^n(S)$ recursively by

$$\mathcal{P}^0(S) = S, \quad \mathcal{P}^{n+1}(S) = \mathcal{P}(\mathcal{P}^n(S)) \quad (n \geq 0).$$

Likewise, define the iterated *nonempty* powersets $(\mathcal{P}^*)^n(S)$ by

$$(\mathcal{P}^*)^0(S) = S, \quad (\mathcal{P}^*)^{n+1}(S) = \mathcal{P}^*((\mathcal{P}^*)^n(S)) \quad (n \geq 0).$$

Informally, an n -SuperHyperGraph is obtained by allowing vertices themselves to be set-valued objects formed by iterating the powerset construction, and then taking hyperedges as nonempty subsets of these higher-level vertices. This layered viewpoint has appeared in several recent works; see, e.g., [6, 7]. Besides their theoretical interest, such hierarchical models have been proposed in applications including molecular modeling, network analysis, and signal processing [27]. Throughout the paper, the level parameter n is always assumed to be a nonnegative integer.

Definition 2.4 (SuperHyperGraph). [5] Fix an integer $n \geq 0$ and a finite *base set* V_0 (whose elements are called *elementary vertices*). Let $\mathcal{P}^n(V_0)$ be the n -fold iterated powerset as in Definition 2.3.

An n -SuperHyperGraph (briefly, an n -SHG) over V_0 is a pair

$$\text{SHG}^{(n)} = (V, E),$$

where

$$V \subseteq \mathcal{P}^n(V_0) \setminus \{\emptyset\} \quad \text{and} \quad E \subseteq \mathcal{P}^*(V).$$

Elements of V are called n -supervertices, and elements of E are called n -superhyperedges (or simply *superedges* when n is clear). We assume V and E are finite.

In particular, when $n = 0$ we have $V \subseteq V_0$ and $E \subseteq \mathcal{P}^*(V)$, so $\text{SHG}^{(0)}$ is an ordinary hypergraph on V .

Remark 2.5 (On ‘‘hierarchy’’). For $n \geq 1$, each n -supervertex $x \in V \subseteq \mathcal{P}^n(V_0)$ is a set whose elements lie in $\mathcal{P}^{n-1}(V_0)$, and hence x can be regarded as a $(n-1)$ -level collection of lower-level objects. Thus the iterated powerset formalism encodes a canonical multi-level nesting.

2.2 Neutrosophic SuperHyperGraphs

We next recall a common single-valued neutrosophic formalism and adapt it to the n -SuperHyperGraph setting (cf. [5]).

Definition 2.6 (Single-valued neutrosophic set). [28] Let X be a set. A *single-valued neutrosophic set* A on X is specified by three functions

$$T_A, I_A, F_A : X \rightarrow [0, 1],$$

called the *truth-membership*, *indeterminacy-membership*, and *falsity-membership* functions, respectively, such that

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \quad (\forall x \in X).$$

The *support* of A is

$$\text{supp}(A) := \{x \in X \mid T_A(x) + I_A(x) + F_A(x) > 0\}.$$

Definition 2.7 (Neutrosophic SuperHyperGraph). [7] Fix $n \geq 0$ and a finite base set V_0 . Let $\text{SHG}^{(n)} = (V, E)$ be an n -SuperHyperGraph over V_0 .

A (*single-valued*) *neutrosophic n -SuperHyperGraph* (briefly, a *neutrosophic n -SHG*) is a tuple

$$\text{NSHG}^{(n)} = (V, E, T_V, I_V, F_V, T_E, I_E, F_E),$$

where

- $T_V, I_V, F_V : V \rightarrow [0, 1]$ assign to each n -supervertex $v \in V$ its truth-, indeterminacy-, and falsity-membership degrees, and satisfy

$$0 \leq T_V(v) + I_V(v) + F_V(v) \leq 3 \quad (\forall v \in V);$$

- $T_E, I_E, F_E : E \times V \rightarrow [0, 1]$ assign to each ordered pair $(e, v) \in E \times V$ an *incidence truth/indeterminacy/falsity triple*, subject to

$$0 \leq T_E(e, v) + I_E(e, v) + F_E(e, v) \leq 3 \quad (\forall e \in E, \forall v \in V),$$

and the *compatibility* and *support* constraints

$$\begin{aligned} T_E(e, v) &\leq T_V(v), & I_E(e, v) &\leq I_V(v), & F_E(e, v) &\leq F_V(v) \\ &(\forall e \in E, \forall v \in V), \\ (T_E(e, v), I_E(e, v), F_E(e, v)) &= (0, 0, 0) && (\forall e \in E, \forall v \in V \setminus e). \end{aligned}$$

In particular, for each $e \in E$, the neutrosophic incidence degrees are supported on the underlying (crisp) superedge membership $v \in e$.

2.3 Spherical Neutrosophic Graph

A spherical neutrosophic graph assigns unit-ball truth, indeterminacy, and falsity degrees to vertices and edges, respecting adjacency constraints [20].

Definition 2.8 (Spherical neutrosophic set). [29, 30] Let X be a nonempty set. A *spherical neutrosophic set* (briefly, an *SNS*) on X is a triple

$$A = (T_A, I_A, F_A),$$

where $T_A, I_A, F_A : X \rightarrow [0, 1]$ are the *truth-, indeterminacy-, and falsity-membership* functions, respectively, such that for every $x \in X$,

$$T_A(x)^2 + I_A(x)^2 + F_A(x)^2 \leq 1.$$

The *support* of A is

$$\text{supp}(A) := \{x \in X : T_A(x)^2 + I_A(x)^2 + F_A(x)^2 > 0\}.$$

Definition 2.9 (Spherical neutrosophic relation). Let V be a nonempty set and let $A = (T_A, I_A, F_A)$ be an SNS on V . A *spherical neutrosophic relation* on V (with respect to A) is a triple

$$B = (T_B, I_B, F_B),$$

where $T_B, I_B, F_B : V \times V \rightarrow [0, 1]$ satisfy, for all $x, y \in V$,

$$\begin{aligned} T_B(x, y)^2 + I_B(x, y)^2 + F_B(x, y)^2 &\leq 1, \\ T_B(x, y) &\leq \min\{T_A(x), T_A(y)\}, \\ I_B(x, y) &\leq \min\{I_A(x), I_A(y)\}, \\ F_B(x, y) &\leq \max\{F_A(x), F_A(y)\}. \end{aligned}$$

If one wishes to model an undirected loopless structure, one additionally imposes

$$\begin{aligned} T_B(x, y) &= T_B(y, x), \quad I_B(x, y) = I_B(y, x), \quad F_B(x, y) = F_B(y, x), \\ T_B(x, x) &= I_B(x, x) = F_B(x, x) = 0. \end{aligned}$$

Definition 2.10 (Spherical neutrosophic graph). Let $G^* = (V, E^*)$ be a finite simple undirected graph. A *spherical neutrosophic graph* (briefly, an *SNG*) on G^* is a pair

$$G = (A, B),$$

where $A = (T_A, I_A, F_A)$ is an SNS on V and $B = (T_B, I_B, F_B)$ is a spherical neutrosophic relation on V (with respect to A) such that

$$(T_B(x, y), I_B(x, y), F_B(x, y)) = (0, 0, 0) \quad \text{whenever } \{x, y\} \notin E^*.$$

Equivalently, the underlying crisp edge set can be recovered as

$$E^* = \left\{ \{x, y\} \subseteq V : x \neq y, (T_B(x, y), I_B(x, y), F_B(x, y)) \neq (0, 0, 0) \right\}.$$

3 Main Results

In this section, we present the main results of this paper.

3.1 Spherical Neutrosophic HyperGraphs

Spherical neutrosophic hypergraphs assign unit-ball truth, indeterminacy, falsity degrees to vertices and hyperedge incidences, consistent with containment.

Definition 3.1 (Spherical neutrosophic hypergraph). Let $\mathcal{H}^* = (V, \mathcal{E})$ be a finite (undirected) hypergraph, i.e., V is a finite nonempty set and $\mathcal{E} \subseteq \mathcal{P}^*(V)$ is a finite family of nonempty subsets of V .

A *spherical neutrosophic hypergraph* (briefly, an *SNHG*) on \mathcal{H}^* is a tuple

$$\mathcal{H} = (V, \mathcal{E}, T_V, I_V, F_V, T_{\mathcal{E}}, I_{\mathcal{E}}, F_{\mathcal{E}}),$$

satisfying:

- **(Spherical neutrosophic vertex set).** (T_V, I_V, F_V) is an SNS on V , i.e., $T_V, I_V, F_V : V \rightarrow [0, 1]$ and

$$T_V(v)^2 + I_V(v)^2 + F_V(v)^2 \leq 1 \quad (\forall v \in V).$$

- **(Spherical neutrosophic incidence degrees).** $T_{\mathcal{E}}, I_{\mathcal{E}}, F_{\mathcal{E}} : \mathcal{E} \times V \rightarrow [0, 1]$ such that

$$T_{\mathcal{E}}(e, v)^2 + I_{\mathcal{E}}(e, v)^2 + F_{\mathcal{E}}(e, v)^2 \leq 1 \quad (\forall e \in \mathcal{E}, \forall v \in V).$$

- **(Support / crisp incidence).** For all $e \in \mathcal{E}$ and $v \in V \setminus e$,

$$(T_{\mathcal{E}}(e, v), I_{\mathcal{E}}(e, v), F_{\mathcal{E}}(e, v)) = (0, 0, 0).$$

- **(Vertex-dominated constraints on incidences).** For all $e \in \mathcal{E}$ and $v \in e$,

$$\begin{aligned} T_{\mathcal{E}}(e, v) &\leq \min_{u \in e} T_V(u), \\ I_{\mathcal{E}}(e, v) &\leq \min_{u \in e} I_V(u), \\ F_{\mathcal{E}}(e, v) &\leq \max_{u \in e} F_V(u). \end{aligned}$$

Remark 3.2 (Why min/max?). When $|e| = 2$, the constraints in Definition 3.1 reduce exactly to the usual spherical-neutrosophic graph constraints $T(\cdot) \leq \min(\cdot)$, $I(\cdot) \leq \min(\cdot)$, $F(\cdot) \leq \max(\cdot)$ used for edges between two vertices.

Theorem 3.3 (SNHGs generalize spherical neutrosophic graphs). *Every spherical neutrosophic graph can be realized as a spherical neutrosophic hypergraph whose hyperedges all have cardinality 2.*

Proof. Let $G = (A, B)$ be a spherical neutrosophic graph on a finite vertex set V , where $A = (T_A, I_A, F_A)$ is the spherical neutrosophic vertex set and $B = (T_B, I_B, F_B)$ is the spherical neutrosophic relation on $V \times V$.

Define a crisp (simple) graph $G^* = (V, E^*)$ by

$$E^* := \left\{ \{x, y\} \subseteq V : x \neq y, (T_B(x, y), I_B(x, y), F_B(x, y)) \neq (0, 0, 0) \right\}.$$

Now view E^* as a 2-uniform hyperedge family $\mathcal{E} := E^* \subseteq \mathcal{P}^*(V)$.

Define vertex-memberships for the hypergraph by

$$T_V := T_A, \quad I_V := I_A, \quad F_V := F_A.$$

Define incidence memberships $T_{\mathcal{E}}, I_{\mathcal{E}}, F_{\mathcal{E}} : \mathcal{E} \times V \rightarrow [0, 1]$ by

$$(T_{\mathcal{E}}(e, v), I_{\mathcal{E}}(e, v), F_{\mathcal{E}}(e, v))$$

$$:= \begin{cases} (T_B(x, y), I_B(x, y), F_B(x, y)) & \text{if } e = \{x, y\} \in \mathcal{E} \text{ and } v \in \{x, y\}, \\ (0, 0, 0) & \text{if } v \notin e. \end{cases}$$

We verify the axioms of Definition 3.1:

(i) *Spherical constraint.* Since A is spherical neutrosophic on V , we have $T_V(v)^2 + I_V(v)^2 + F_V(v)^2 \leq 1$ for all $v \in V$. Also, since B is spherical neutrosophic on $V \times V$, each triple $(T_B(x, y), I_B(x, y), F_B(x, y))$ satisfies the spherical constraint; hence so does $(T_{\mathcal{E}}(e, v), I_{\mathcal{E}}(e, v), F_{\mathcal{E}}(e, v))$ for every (e, v) .

(ii) *Support.* By construction, $(T_{\mathcal{E}}(e, v), I_{\mathcal{E}}(e, v), F_{\mathcal{E}}(e, v)) = (0, 0, 0)$ whenever $v \notin e$.

(iii) *Vertex-dominated inequalities.* Fix $e = \{x, y\} \in \mathcal{E}$ and $v \in e$. From the defining inequalities of a spherical neutrosophic graph,

$$T_B(x, y) \leq \min\{T_A(x), T_A(y)\},$$

$$I_B(x, y) \leq \min\{I_A(x), I_A(y)\},$$

$$F_B(x, y) \leq \max\{F_A(x), F_A(y)\}.$$

Thus, using $T_V = T_A$, $I_V = I_A$, $F_V = F_A$ and $\min_{u \in e} = \min\{\cdot, \cdot\}$, $\max_{u \in e} = \max\{\cdot, \cdot\}$, we obtain

$$T_{\mathcal{E}}(e, v) \leq \min_{u \in e} T_V(u), \quad I_{\mathcal{E}}(e, v) \leq \min_{u \in e} I_V(u), \quad F_{\mathcal{E}}(e, v) \leq \max_{u \in e} F_V(u).$$

Therefore $\mathcal{H} := (V, \mathcal{E}, T_V, I_V, F_V, T_{\mathcal{E}}, I_{\mathcal{E}}, F_{\mathcal{E}})$ is an SNHG. Moreover, the original graph data are recovered on each edge $\{x, y\}$ by the constant incidence assignment above, so G is realized as a special case of an SNHG. \square

3.2 Spherical Neutrosophic SuperHyperGraph

A spherical neutrosophic n -SuperHyperGraph assigns unit-ball neutrosophic memberships to n -supervertices and superedge incidences, respecting crisp containment constraints.

Definition 3.4 (Spherical neutrosophic SuperHyperGraph). Let $\text{SHG}^{(n)} = (V, E)$ be a crisp n -SuperHyperGraph over a finite base set V_0 . A *spherical neutrosophic n -SuperHyperGraph* (briefly, an SNSHG_n) is a tuple

$$\mathfrak{G}^{(n)} = (V, E, T_V, I_V, F_V, T_E, I_E, F_E)$$

such that:

- **(Spherical supervertex-memberships).** $T_V, I_V, F_V : V \rightarrow [0, 1]$ and

$$T_V(v)^2 + I_V(v)^2 + F_V(v)^2 \leq 1 \quad (\forall v \in V).$$

- **(Spherical incidence-memberships).** $T_E, I_E, F_E : E \times V \rightarrow [0, 1]$ and

$$T_E(e, v)^2 + I_E(e, v)^2 + F_E(e, v)^2 \leq 1 \quad (\forall e \in E, \forall v \in V).$$

- **(Support / crisp incidence).** For all $e \in E$ and $v \in V \setminus e$,

$$(T_E(e, v), I_E(e, v), F_E(e, v)) = (0, 0, 0).$$

- **(Supervertex-dominated constraints).** For all $e \in E$ and $v \in e$,

$$T_E(e, v) \leq \min_{u \in e} T_V(u),$$

$$I_E(e, v) \leq \min_{u \in e} I_V(u),$$

$$F_E(e, v) \leq \max_{u \in e} F_V(u).$$

The theorem is stated below.

Theorem 3.5 (SNSHG₀s generalize SNHG₀s and SNG₀s).

1. *Every spherical neutrosophic hypergraph is a spherical neutrosophic 0-SuperHyperGraph.*
2. *Consequently, spherical neutrosophic n-SuperHyperGraphs generalize both spherical neutrosophic hypergraphs and spherical neutrosophic graphs.*

Proof. **(1)** Let $\mathcal{H} = (V, \mathcal{E}, T_V, I_V, F_V, T_{\mathcal{E}}, I_{\mathcal{E}}, F_{\mathcal{E}})$ be an SNHG (Definition 3.1). Set $V_0 := V$ and $n := 0$. Then $\mathcal{P}^0(V_0) = V_0 = V$, and (V, \mathcal{E}) is a crisp 0-SuperHyperGraph since $\mathcal{E} \subseteq \mathcal{P}^*(V)$. Define $E := \mathcal{E}$ and set the SNSHG data by

$$T_E := T_{\mathcal{E}}, \quad I_E := I_{\mathcal{E}}, \quad F_E := F_{\mathcal{E}}.$$

All axioms in Definition 3.4 coincide verbatim with those of Definition 3.1 (just with E in place of \mathcal{E}). Hence $\mathfrak{G}^{(0)} = (V, E, T_V, I_V, F_V, T_E, I_E, F_E)$ is an SNSHG₀.

(2) By Theorem 3.3, every spherical neutrosophic graph is a special case of an SNHG. By part (1), every SNHG is a special case of an SNSHG₀. Therefore, spherical neutrosophic n -SuperHyperGraphs (in particular $n = 0$) generalize both spherical neutrosophic hypergraphs and spherical neutrosophic graphs. \square

4 Additional Results: Spherical Complex Neutrosophic SuperHyperGraph

In this section, as an additional result, we investigate spherical complex neutrosophic superhypergraphs.

4.1 Complex spherical neutrosophic graph (CSNG)

A Complex Spherical Neutrosophic Graph assigns complex truth–indeterminacy–falsity memberships to vertices and edges, satisfying spherical constraints and adjacency dominance.

Definition 4.1 (Complex spherical neutrosophic number (CSNN)). A *complex spherical neutrosophic number* (CSNN) is a triple

$$\mathbf{u} = (Te^{i2\pi\alpha}, Ie^{i2\pi\beta}, Fe^{i2\pi\gamma}) \in \mathbb{C}^3$$

with

$$T, I, F \in [0, 1], \quad \alpha, \beta, \gamma \in [0, 1],$$

satisfying the spherical constraints

$$T^2 + I^2 + F^2 \leq 1, \quad \alpha^2 + \beta^2 + \gamma^2 \leq 1.$$

Definition 4.2 (Complex spherical neutrosophic set (CSNS)). Let X be a nonempty set. A *complex spherical neutrosophic set* (CSNS) on X is a map

$$N : X \rightarrow \text{CSNN}, \quad x \mapsto N(x) = (T_N(x)e^{i2\pi\alpha_N(x)}, I_N(x)e^{i2\pi\beta_N(x)}, F_N(x)e^{i2\pi\gamma_N(x)}),$$

such that for all $x \in X$,

$$T_N(x)^2 + I_N(x)^2 + F_N(x)^2 \leq 1, \quad \alpha_N(x)^2 + \beta_N(x)^2 + \gamma_N(x)^2 \leq 1.$$

Definition 4.3 (Complex spherical neutrosophic relation). Let V be a nonempty set and let N be a CSNS on V . A *complex spherical neutrosophic relation* on V (with respect to N) is a map

$$S : V \times V \rightarrow \text{CSNN},$$

with

$$S(x, y) = (T_S(x, y)e^{i2\pi\alpha_S(x, y)}, I_S(x, y)e^{i2\pi\beta_S(x, y)}, F_S(x, y)e^{i2\pi\gamma_S(x, y)}),$$

such that for all $x, y \in V$:

(i) **(Spherical constraint).**

$$T_S(x, y)^2 + I_S(x, y)^2 + F_S(x, y)^2 \leq 1, \quad \alpha_S(x, y)^2 + \beta_S(x, y)^2 + \gamma_S(x, y)^2 \leq 1.$$

(ii) **(Endpoint-dominance, neutrosophic-style).**

$$T_S(x, y) \leq \min\{T_N(x), T_N(y)\}, \quad I_S(x, y) \leq \min\{I_N(x), I_N(y)\},$$

$$F_S(x, y) \leq \max\{F_N(x), F_N(y)\},$$

and analogously for normalized phases

$$\alpha_S(x, y) \leq \min\{\alpha_N(x), \alpha_N(y)\}, \quad \beta_S(x, y) \leq \min\{\beta_N(x), \beta_N(y)\},$$

$$\gamma_S(x, y) \leq \max\{\gamma_N(x), \gamma_N(y)\}.$$

For an undirected loopless setting, additionally impose

$$S(x, y) = S(y, x), \quad S(x, x) = (0, 0, 0) \quad (\forall x, y \in V).$$

Definition 4.4 (Complex spherical neutrosophic graph (CSNG)). Let $G^* = (V, E^*)$ be a finite simple undirected graph. A *complex spherical neutrosophic graph* (CSNG) on G^* is a pair

$$\mathcal{G} = (N, S),$$

where N is a CSNS on V and S is a complex spherical neutrosophic relation on V (with respect to N), such that

$$S(x, y) = (0, 0, 0) \quad \text{whenever } \{x, y\} \notin E^*.$$

Proposition 4.5 (Well-definedness of Definition 4.4). *Definition 4.4 is well-defined in the sense that:*

- (i) *the condition $S(x, y) = (0, 0, 0)$ is unambiguous within the codomain of complex spherical neutrosophic numbers;*
- (ii) *for any pair (N, S) satisfying the stated axioms, the induced (support) edge set*

$$E_S := \left\{ \{x, y\} \subseteq V : x \neq y, S(x, y) \neq (0, 0, 0) \right\}$$
is a well-defined simple undirected graph and satisfies $E_S \subseteq E^$;*
- (iii) *hence the phrase “a CSNG on G^* ” defines a mathematically consistent class of objects.*

Proof. (i) The zero triple is a valid complex spherical neutrosophic number.

Recall that a complex spherical neutrosophic number (CSNN) has the form

$$(Te^{i2\pi\alpha}, Ie^{i2\pi\beta}, Fe^{i2\pi\gamma}) \in \mathbb{C}^3$$

with $T, I, F \in [0, 1]$, $\alpha, \beta, \gamma \in [0, 1]$, and the spherical constraints

$$T^2 + I^2 + F^2 \leq 1, \quad \alpha^2 + \beta^2 + \gamma^2 \leq 1.$$

Setting $T = I = F = 0$ yields the complex triple $(0, 0, 0) \in \mathbb{C}^3$, which satisfies $0^2 + 0^2 + 0^2 \leq 1$. Thus $(0, 0, 0)$ belongs to the CSNN codomain.

Moreover, if one insists on the polar representation, note that for any $\theta \in \mathbb{R}$, $0 \cdot e^{i\theta} = 0$ as a complex number. Hence the value $(0, 0, 0)$ is a *unique* element of \mathbb{C}^3 independent of any phase parameters; therefore the statement $S(x, y) = (0, 0, 0)$ is unambiguous.

(ii) The support constraint is coherent and yields a well-defined induced crisp graph. By assumption, $S : V \times V \rightarrow \text{CSNN} \subseteq \mathbb{C}^3$ is a complex spherical

neutrosophic relation on V (with respect to N). In particular, for every $(x, y) \in V \times V$ the value $S(x, y)$ is a well-defined element of \mathbb{C}^3 , so the predicate “ $S(x, y) \neq (0, 0, 0)$ ” is well-defined. Hence the set

$$E_S := \left\{ \{x, y\} \subseteq V : x \neq y, S(x, y) \neq (0, 0, 0) \right\}$$

is well-defined.

If, as in the usual *undirected loopless* convention for such relations, S is symmetric and $S(x, x) = (0, 0, 0)$ for all $x \in V$, then E_S is an edge set of a simple undirected graph on V (no loops by the condition $x \neq y$, and no orientations because $\{x, y\}$ is an unordered pair).

Finally, by the support axiom in Definition 4.4,

$$S(x, y) = (0, 0, 0) \quad \text{whenever } \{x, y\} \notin E^*,$$

so if $\{x, y\} \in E_S$ then necessarily $\{x, y\} \in E^*$; equivalently, $E_S \subseteq E^*$.

(iii) Consistency of the class of objects. Parts (i)–(ii) show that every pair (N, S) satisfying Definition 4.4 is a legitimate mathematical structure: the codomains are correct, the zero condition is meaningful, and the support restriction is compatible with the fixed crisp graph G^* (indeed it forces $E_S \subseteq E^*$). Therefore Definition 4.4 specifies a consistent and well-defined notion of a complex spherical neutrosophic graph on G^* . \square

4.2 Complex spherical neutrosophic SuperHyperGraph

A complex spherical neutrosophic SuperHyperGraph assigns complex-valued truth–indeterminacy–falsity memberships to supervertices and superedge incidences under spherical constraints.

Definition 4.6 (Complex spherical neutrosophic n -SuperHyperGraph). Fix a finite base set V_0 and an integer $n \geq 0$. Let $\text{SHG}^{(n)} = (V, E)$ be a *crisp* n -SuperHyperGraph over V_0 , i.e., $V \subseteq \mathcal{P}^n(V_0) \setminus \{\emptyset\}$ and $E \subseteq \mathcal{P}^*(V)$.

A *complex spherical neutrosophic n -SuperHyperGraph* (briefly, a CSNSHG_n) on $\text{SHG}^{(n)}$ is a tuple

$$\mathfrak{G}_{\mathbb{C}}^{(n)} = (V, E, T_V, I_V, F_V, \alpha_V, \beta_V, \gamma_V, T_E, I_E, F_E, \alpha_E, \beta_E, \gamma_E)$$

where the following conditions hold.

- (i) **(Complex spherical neutrosophic supervertices).** For each $v \in V$, define the complex neutrosophic triple

$$N_V(v) := (T_V(v)e^{i2\pi\alpha_V(v)}, I_V(v)e^{i2\pi\beta_V(v)}, F_V(v)e^{i2\pi\gamma_V(v)}) \in \mathbb{C}^3,$$

where

$$T_V, I_V, F_V : V \rightarrow [0, 1], \quad \alpha_V, \beta_V, \gamma_V : V \rightarrow [0, 1],$$

and the *spherical constraints* hold for all $v \in V$:

$$T_V(v)^2 + I_V(v)^2 + F_V(v)^2 \leq 1, \quad \alpha_V(v)^2 + \beta_V(v)^2 + \gamma_V(v)^2 \leq 1.$$

(ii) **(Complex spherical neutrosophic superedge incidences).** For each $(e, v) \in E \times V$, define the complex incidence triple

$$N_E(e, v) := (T_E(e, v)e^{i2\pi\alpha_E(e, v)}, I_E(e, v)e^{i2\pi\beta_E(e, v)}, F_E(e, v)e^{i2\pi\gamma_E(e, v)}) \in \mathbb{C}^3,$$

where

$$T_E, I_E, F_E : E \times V \rightarrow [0, 1], \quad \alpha_E, \beta_E, \gamma_E : E \times V \rightarrow [0, 1],$$

and for all $(e, v) \in E \times V$,

$$T_E(e, v)^2 + I_E(e, v)^2 + F_E(e, v)^2 \leq 1, \quad \alpha_E(e, v)^2 + \beta_E(e, v)^2 + \gamma_E(e, v)^2 \leq 1.$$

(iii) **(Support / crisp incidence).** For all $e \in E$ and $v \in V \setminus e$,

$$N_E(e, v) = (0, 0, 0), \quad \text{i.e.,} \quad T_E(e, v) = I_E(e, v) = F_E(e, v) = 0 \quad (\text{phases irrelevant}).$$

(iv) **(Supervertex-dominance: amplitudes).** For all $e \in E$ and $v \in e$,

$$T_E(e, v) \leq \min_{u \in e} T_V(u), \quad I_E(e, v) \leq \min_{u \in e} I_V(u),$$

$$F_E(e, v) \leq \max_{u \in e} F_V(u).$$

(v) **(Supervertex-dominance: phases).** For all $e \in E$ and $v \in e$,

$$\alpha_E(e, v) \leq \min_{u \in e} \alpha_V(u), \quad \beta_E(e, v) \leq \min_{u \in e} \beta_V(u),$$

$$\gamma_E(e, v) \leq \max_{u \in e} \gamma_V(u).$$

(If one prefers angle parameters in $[0, 2\pi]$, replace $\alpha, \beta, \gamma \in [0, 1]$ by $\theta_\alpha, \theta_\beta, \theta_\gamma \in [0, 2\pi]$ and set $\alpha = \theta_\alpha/(2\pi)$, etc.)

Theorem 4.7 (CSNSHG_n generalizes CSNG and spherical neutrosophic SHG).

(1) **(Generalizes CSNG).** Every complex spherical neutrosophic graph (CSNG) is realizable as a CSNSHG₀ whose superedges all have cardinality 2.

- (2) (**Generalizes spherical neutrosophic n -SuperHyperGraphs**). Every (real) spherical neutrosophic n -SuperHyperGraph is obtained as a special case of a CSNSHG $_n$ by taking all phase functions identically zero.
- (3) (**Consequently**). For each $n \geq 0$, CSNSHG $_n$ simultaneously generalizes CSNG and spherical neutrosophic n -SuperHyperGraphs.

Proof. **(1) Embedding of CSNG into CSNSHG $_0$ with 2-uniform edges.** Let $\mathcal{G} = (N, S)$ be a CSNG on a finite simple graph $G^* = (V, E^*)$. Write

$$N(x) = (T_N(x)e^{i2\pi\alpha_N(x)}, I_N(x)e^{i2\pi\beta_N(x)}, F_N(x)e^{i2\pi\gamma_N(x)}),$$

and

$$S(x, y) = (T_S(x, y)e^{i2\pi\alpha_S(x, y)}, I_S(x, y)e^{i2\pi\beta_S(x, y)}, F_S(x, y)e^{i2\pi\gamma_S(x, y)}),$$

where $S(x, y) = (0, 0, 0)$ whenever $\{x, y\} \notin E^*$, and the CSNG axioms include the spherical constraints and endpoint-dominance inequalities.

Define $V_0 := V$ and $n := 0$. Then $\mathcal{P}^0(V_0) = V$, and (V, E) with

$$E := E^* \subseteq \binom{V}{2} \subseteq \mathcal{P}^*(V)$$

is a crisp 0-SuperHyperGraph (indeed a 2-uniform hypergraph).

Now define the CSNSHG $_0$ data as follows:

$$T_V := T_N, \quad I_V := I_N, \quad F_V := F_N, \quad \alpha_V := \alpha_N, \quad \beta_V := \beta_N, \quad \gamma_V := \gamma_N.$$

For incidence parameters, set for each $e = \{x, y\} \in E$ and each $v \in V$,

$$(T_E(e, v), I_E(e, v), F_E(e, v), \alpha_E(e, v), \beta_E(e, v), \gamma_E(e, v)) := \begin{cases} (T_S(x, y), I_S(x, y), F_S(x, y), \alpha_S(x, y), \beta_S(x, y), \gamma_S(x, y)) & \text{if } v \in \{x, y\}, \\ (0, 0, 0, 0, 0, 0) & \text{if } v \notin e. \end{cases}$$

We verify Definition 4.6:

Spherical constraints. They hold on vertices because N is a CSNS on V , and on incidences because S is a complex spherical neutrosophic relation on $V \times V$.

Support. If $v \notin e$, then by construction $N_E(e, v) = (0, 0, 0)$.

Dominance (amplitudes and phases). Fix $e = \{x, y\}$ and $v \in e$. The CSNG endpoint-dominance inequalities give

$$T_S(x, y) \leq \min\{T_N(x), T_N(y)\},$$

$$I_S(x, y) \leq \min\{I_N(x), I_N(y)\},$$

$$F_S(x, y) \leq \max\{F_N(x), F_N(y)\},$$

and similarly for phases

$$\begin{aligned}\alpha_S(x, y) &\leq \min\{\alpha_N(x), \alpha_N(y)\}, \\ \beta_S(x, y) &\leq \min\{\beta_N(x), \beta_N(y)\}, \\ \gamma_S(x, y) &\leq \max\{\gamma_N(x), \gamma_N(y)\}.\end{aligned}$$

Since $\min_{u \in e}$ and $\max_{u \in e}$ coincide with $\min\{\cdot, \cdot\}$ and $\max\{\cdot, \cdot\}$ for $e = \{x, y\}$, these are exactly the dominance conditions required in Definition 4.6. Hence we obtain a CSNSHG₀ whose superedges all have size 2, realizing the original CSNG.

(2) Embedding of spherical neutrosophic n -SuperHyperGraphs (real) into CSNSHG _{n} . Let

$$\mathfrak{G}^{(n)} = (V, E, T_V, I_V, F_V, T_E, I_E, F_E)$$

be a (real) spherical neutrosophic n -SuperHyperGraph, i.e.,

$$T_V(v)^2 + I_V(v)^2 + F_V(v)^2 \leq 1, \quad T_E(e, v)^2 + I_E(e, v)^2 + F_E(e, v)^2 \leq 1,$$

together with the usual support and min/min/max dominance constraints.

Define a complex structure by setting *all phases to zero*:

$$\begin{aligned}\alpha_V &= \beta_V = \gamma_V \equiv 0 \text{ on } V, \\ \alpha_E &= \beta_E = \gamma_E \equiv 0 \text{ on } E \times V,\end{aligned}$$

and interpret memberships as complex numbers with argument 0:

$$\begin{aligned}N_V(v) &:= (T_V(v)e^{i0}, I_V(v)e^{i0}, F_V(v)e^{i0}), \\ N_E(e, v) &:= (T_E(e, v)e^{i0}, I_E(e, v)e^{i0}, F_E(e, v)e^{i0}).\end{aligned}$$

Then the *phase spherical constraints* hold trivially because $0^2 + 0^2 + 0^2 = 0 \leq 1$, while the amplitude spherical constraints, support constraints, and dominance constraints coincide exactly with those of the given spherical neutrosophic n -SuperHyperGraph. Therefore the resulting tuple satisfies Definition 4.6, so it is a CSNSHG _{n} .

Parts (1) and (2) establish that CSNSHG _{n} contains CSNG (via $n = 0$ and 2-uniform superedges) and contains spherical neutrosophic n -SuperHyperGraphs (via zero phases) as special cases. Hence CSNSHG _{n} generalizes both frameworks. \square

5 Open Problems

Finally, we list several questions that may be regarded as open problems.

Open Problem 5.1 (Plithogenic and advanced neutrosophic extensions). Develop a systematic theory of *plithogenic* and other advanced neutrosophic enrichments of spherical neutrosophic hypergraphs and spherical neutrosophic n -superhypergraphs. In particular:

- (i) Define plithogenic analogues of spherical neutrosophic (super)hypergraphs by introducing a contradiction (or appurtenance) parameter in the sense of Plithogenic sets [31, 32, 17].
- (ii) Formulate and study corresponding versions based on SuperHyperNeutrosophic sets [33].
- (iii) Incorporate neutrosophic offsets [34, 35] and analyze how offset operations interact with spherical constraints, incidence dominance conditions, and structural invariants.

Establish basic closure properties, canonical constructions, and meaningful invariants (e.g., degrees, connectivity, colorings, and index-like quantities) in these enriched settings.

Open Problem 5.2 (Computation, data-driven validation, and software). Design computational pipelines for spherical neutrosophic (super)hypergraph models, and validate them on real-world data. Specifically:

- (i) Identify representative application domains and datasets where higher-order and hierarchical uncertainty are natural (e.g., group interactions, multi-modal relations, layered systems).
- (ii) Develop algorithms for core tasks (construction, incidence estimation, parameter learning, visualization, and evaluation) under spherical constraints.
- (iii) Implement an open, reusable software library that supports modeling, computation, and visualization for spherical neutrosophic graphs, hypergraphs, and n -superhypergraphs, including standardized input/output formats and benchmark instances.

Provide empirical comparisons against classical and fuzzy/neutrosophic baselines to clarify when the proposed models offer measurable advantages.

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